AP Calculus AB
Section 6.4 – Riemann Sums and Definite Integrals Notesheet

Name: Key
Date: ________ Block: ____

How can rectangular approximation methods become more accurate?

Conversion from Riemann Sum Notation to Integral Notation

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x = \int_a^b f(x)dx$$
$$\Delta x = \frac{b-a}{n}$$
$$x_i = a + i\cdot\Delta x$$

Converting from Integral Notation to Riemann Sum Notation

1.) $\int_0^3 x^3 dx$ $\quad \Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$

$x_i = a + i\Delta x$ (1st x)

$x_i = a + \frac{3i}{n} = 0 + \frac{3i}{n} = \frac{3i}{n}$

$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$

$\lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\cdot\frac{3}{n} \Rightarrow \boxed{\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{3i}{n}\right)^3\cdot\frac{3}{n}}$

2.) $\int_{-2}^{3}(1-5x^2)dx$

$\Delta x = \frac{b-a}{n} = \frac{3--2}{n} = \frac{5}{n}$

$x_i = a + i\Delta x$

$x_i = -2 + i\left(\frac{5}{n}\right) = -2 + \frac{5i}{n}$

$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-2+\frac{5i}{n}\right)\left(\frac{5}{n}\right) \Rightarrow \boxed{\lim_{n\to\infty}\sum_{i=1}^{n}\left[1-5\left(-2+\frac{5i}{n}\right)^2\right]\frac{5}{n}}$

3.) $\int_{\pi}^{2\pi}\cos(x)\,dx$

$\Delta x = \frac{b-a}{n} = \frac{2\pi-\pi}{n} = \frac{\pi}{n}$

$x_i = a + i\Delta x$

$x_i = \pi + \frac{\pi i}{n}$

$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\pi+\frac{\pi i}{n}\right)\frac{\pi}{n} \Rightarrow \boxed{\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{n}\cos\left(\pi+\frac{\pi i}{n}\right)}$

Converting from Riemann Sum Notation to Integral Notation

1.) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(4\left(\frac{5i}{n}\right)\right)\cdot\frac{5}{n}$

$\Delta x = \frac{5}{n}$ $\quad \frac{5}{n} = \frac{b-a}{n}$ $\quad 5 = b - a$

$x_i = \frac{5i}{n}$ $\quad a + i\Delta x = x_i$

$a + \frac{5i}{n} = \frac{5i}{n}$ $\quad a = 0$

$5 = b - a$

$5 = b$

$\boxed{\int_0^5 4x\,dx}$

2.) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(2\left(4+\frac{2i}{n}\right)^2\right)\cdot\frac{2}{n}$

$\Delta x = \frac{2}{n}$ $\quad \frac{2}{n} = \frac{b-a}{n}$ $\quad 2 = b - a$

$2 = b - a$

$2 = b - 4$

$6 = b$

$\boxed{\int_4^6 2x^2 dx}$

$x_i = 4 + \frac{2i}{n}$ $\quad a + i\Delta x = x_i$

$a + \frac{2i}{n} = 4 + \frac{2i}{n}$ $\quad a = 4$

3.) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\left(\frac{4i}{n}-5\right)^2 + 3\left(\frac{4i}{n}-5\right)+5\right)\cdot\frac{4}{n}$

$\boxed{\int_{-5}^{-1}(x^2+3x+5)dx}$

$\Delta x = \frac{4}{n}$ $\quad x_i = a + i\Delta x$

$x_i = \frac{4i}{n} - 5$ $\quad \frac{4i}{n} - 5 = a + \frac{4i}{n}$ $\quad a = -5$

$\frac{b-a}{n} = \frac{4}{n}$

$b - a = 4$ $\quad b - -5 = 4$

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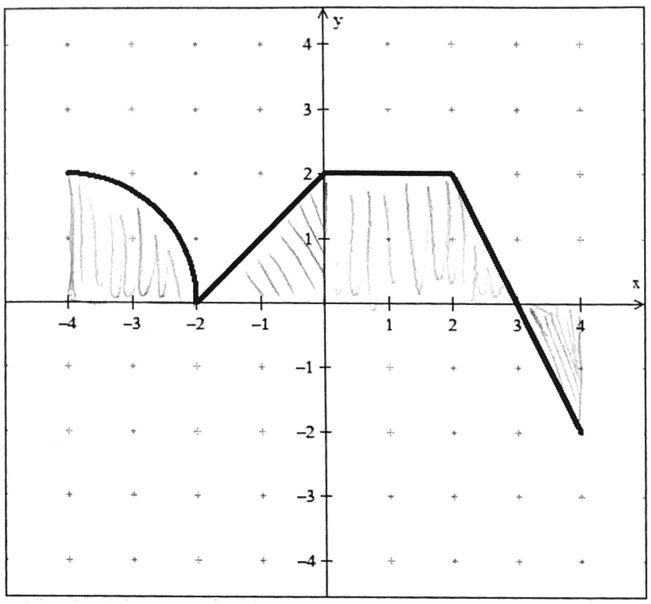
We do not need to evaluate the complicated Riemann Sum Notation as in the examples above, but how can we determine the exact value of the integral without using approximations?

Method 1: Use Area Formulas



Note: Area below the x – axis is negative!

1.) $\int_{-4}^{-2} f(x)dx$ $\frac{1}{4}\pi r^2 = \frac{1}{4}\pi(2)^2 = \pi$

2.) $\int_{-2}^{2} f(x)dx$ $= \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$

$\frac{1}{2}(2)(2) + 2(2) = 2 + 4 = 6$

3.) $\int_{2}^{4} f(x)dx$ $\int_{2}^{3} f(x)\,dx + \int_{3}^{4} f(x)\,dx$

$\frac{1}{2}(1)(2) + \frac{1}{2}(1)(-2) = 1 - 1 = 0$

4.) $\int_{2}^{4} |f(x)|dx$

$\int_{2}^{3} f(x)dx + \int_{3}^{4} f(x)dx$

$\frac{1}{2}(1)(2) + \left|\frac{1}{2}(1)(-2)\right|$

$1 + 1 = 2$

Method 2: Using Properties of Integrals

I. Addition Property:

If $a < b < c$, then $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$

II. Coefficient Property:

For any real Number c, $\int_a^b cf(x)dx = c\int_a^b f(x)dx$

III. Bounds Property:

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

IV. Integral Sum/Difference Property:

$\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

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Examples: Using the given and the definite integral properties, solve the following.

1. Given: $\int_0^2 f(x)dx = 5$ $\int_2^4 f(x)dx = 3$ $\int_2^6 f(x)dx = 12$

a.) $\int_0^6 f(x)dx =$

$\int_0^2 f(x)\,dx + \int_2^6 f(x)\,dx$

$5 + 12$

17

b.) $\int_6^2 f(x)dx =$

$-\int_2^6 f(x)\,dx$

-12

c.) $\int_0^2 4f(x)dx =$

$4\int_0^2 f(x)\,dx$

$4(5) = 20$

d.) $\int_6^0 2f(x)dx =$

$-2\int_0^6 f(x)\,dx$

$-2(17) = -34$

e.) $\int_4^6 [f(x) + 2]dx =$

$\int_4^6 f(x)\,dx + \int_4^6 2\,dx$

$\int_2^6 f(x)\,dx - \int_0^2 f(x)\,dx + 2x\Big]_4^6$

$12 - 5 + [12 - 8] \Rightarrow 7 + 4 = 11$

f.) $\int_4^0 -f(x)dx =$

$\int_0^4 f(x)\,dx$

$\int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx$

$5 + 3 = 8$

2. Given: $\int_1^4 f(x)dx = 6$ $\int_1^4 g(x)dx = 3$

a.) $\int_1^4 [3f(x) + 7g(x) + 2]dx$

$3\int_1^4 f(x)\,dx + 7\int_1^4 g(x)\,dx + \int_1^4 2\,dx$

$3(6) + 7(3) + 2x\Big]_1^4$

$18 + 21 + [8 - 2]$

$39 + 6$

45

b.) $\int_4^1 [2g(x) - f(x) - 3]\,dx$

$-2\int_1^4 g(x)\,dx + \int_1^4 f(x)\,dx + \int_1^4 3\,dx$

$-2(3) + 6 + 3x\Big]_1^4$

$-6 + 6 + [12 - 3]$

9

8. Which of the following limits is equal to $\int_3^5 x^4\,dx$? $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3+\frac{2k}{n}\right)^4\left(\frac{2}{n}\right)$

(A) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3+\frac{k}{n}\right)^4\frac{1}{n}$

(B) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3+\frac{k}{n}\right)^4\frac{2}{n}$

(C) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3+\frac{2k}{n}\right)^4\frac{1}{n}$

(D) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3+\frac{2k}{n}\right)^4\frac{2}{n}$

$\frac{b-a}{n}=\frac{5-3}{n}=\frac{2}{n}$

So we just need to stress that $\Delta x=\frac{b-a}{n}$, that $x_k=a+k\Delta x$ and that you need to substitute x_k into $f(x)$

So

$$\int_a^b f(x)\,dx=\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$$

So the answer to the question above is D.)

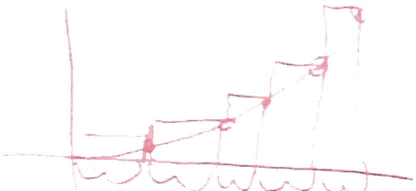
$$\Delta x=\frac{5-3}{n}=\frac{2}{n}$$

$$x_k=3+\frac{2}{n}k$$

$$f(x_k)=\left(3+\frac{2k}{n}\right)^4$$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(3+\frac{2k}{n}\right)^4\cdot\frac{2}{n}$$

My plan is for on Tuesday next week to start class with this and then move into definite integrals using area formulas under the curve and properties of integrals as well. What do you guys think?



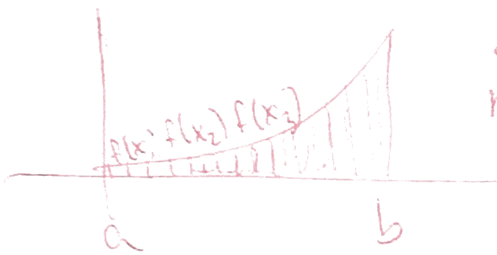
$\Delta x = \frac{b-a}{n} \Delta x$

$\lim_{n\to\infty} \frac{b-a}{n} = \Delta x$

Δx is essentially 0

$A = bh$

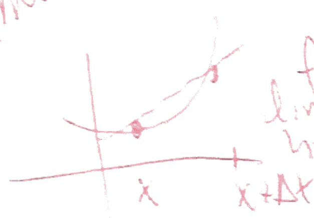
$= \underbrace{f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x + f(x_5)\Delta}_{\infty \text{ many}}$



$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ ← Greek

$x_i = a + i\Delta x$ ← 1Δx then 2Δx etc

similar



$\lim_{h\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

← english replaced

$\int_a^b f(x)\,dx$

$\int_0^3 x^3\,dx$

$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$

$x_i = a + i\Delta x = 0 + i\left(\frac{3}{n}\right) = \frac{3i}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3i}{n}\right)^3 \cdot \frac{3}{n}$

AP Calculus AB
Section 6.3 - Worksheet

Name: ______________________
Date: ______________ Block: __________

Convert the following from Integral Notation to Riemann Sum Notation:

1. $\int_0^5 (x^4+2)dx$

$\Delta x = \frac{b-a}{n} = \frac{5-0}{n} = \frac{5}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ $\quad x_i = a + i\Delta x$

$x_i = 0 + \frac{5i}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{5i}{n}\right)\frac{5}{n}$ $\quad x_i = \frac{5i}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{5i}{n}\right)^4 + 2\right]\frac{5}{n}$

2. $\int_{-1}^5 (x^2-x-1)dx$ $\quad \Delta x = \frac{5--1}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(-1+\frac{6i}{n}\right)\frac{6}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(-1+\frac{6i}{n}\right)^2 - \left(-1+\frac{6i}{n}\right) - 1\right]\frac{6}{n}$

3. $\int_{-2}^3 \frac{1}{x^2+1}dx$ $\quad \Delta x = \frac{3--2}{n} = \frac{5}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(-2+\frac{5i}{n}\right)\left(\frac{5}{n}\right)$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{\left(-2+\frac{5i}{n}\right)^2+1}\left(\frac{5}{n}\right)$

Convert the following from Riemann Sum Notation to Integral Notation:

1. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(3\left(\frac{4i}{n}\right)\right)\cdot\frac{4}{n}$

$\Delta x = \frac{b-a}{n} = \frac{4}{n}$

$a = 0$ $\quad \frac{b-0}{n} = \frac{4}{n}$

$b = 4$

$\boxed{\int_0^4 3x\,dx}$

2. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{2i}{n}-1\right)^3 - 2\left(\frac{2i}{n}-1\right)^2 + 3\right)\cdot\frac{2}{n}$

$\Delta x = \frac{b-a}{n} = \frac{2}{n}$ $\quad \frac{b--1}{n} = \frac{2}{n}$

$a = -1$ $\quad b+1 = 2$

$b = 1$

$\boxed{\int_{-1}^1 (x^3 - 2x^2 + 3)dx}$

3. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\sin\left(2\pi + \frac{3\pi i}{2n}\right)\right)\cdot\frac{3\pi}{2n}$

$\frac{b-a}{n} = \frac{3\pi}{2n}$ $\quad \frac{b-2\pi}{n} = \frac{3\pi}{2n}$ $\quad 2b = 7\pi$

$b = \frac{7\pi}{2}$

$a = 2\pi$ $\quad 2n(b-2\pi) = 3\pi n$

$2b - 4\pi = 3\pi$

$\boxed{\int_{2\pi}^{\frac{7\pi}{2}} \sin x\,dx}$

Given: $\quad \int_0^1 f(x)dx = 2 \quad \int_1^2 f(x)dx = 3 \quad \int_0^1 g(x)dx = -1 \quad \int_0^2 g(x)dx = 4$

1. $\int_0^2 f(x)dx =$

$\int_0^1 f(x)dx + \int_1^2 f(x)dx$

$2 + 3 = 5$

2. $\int_0^2 [f(x)+2g(x)]dx =$

$\int_0^2 f(x)dx + 2\int_0^2 g(x)dx$

$5 + 2(4) = 13$

3. $\int_1^2 g(x)dx =$

$\int_0^1 g(x)dx + \int_1^2 g(x)dx = \int_0^2 g(x)dx$

$-1 + \int_1^2 g(x)dx = 4$

$\int_1^2 g(x)dx = 5$

4. $\int_2^0 g(x)dx =$

$-\int_0^2 g(x)dx = -4$

5. $\int_2^1 3f(x)dx =$

$-3\int_1^2 f(x)dx$

$-3(3) = -9$

6. $\int_2^1 [g(x)-2f(x)]dx =$

$-\int_1^2 g(x)dx + 2\int_1^2 f(x)dx$

$-5 + 2(3) = 1$

7. $\int_0^2 [2f(x)-3g(x)]dx =$

$2\int_0^2 f(x)dx - 3\int_0^2 g(x)dx$

$2(5) - 3(4)$

$10 - 12 = -2$

8. $\int_0^1 [2f(x)+3g(x)-4]dx =$

$2\int_0^1 f(x)dx + 3\int_0^1 g(x)dx - \int_0^1 4\,dx$

$2(2) + 3(-1) - [4 - 4(0)]\ 4x]_0^1$

$4 - 3 - 4 = -3$

9. $\int_1^2 f(x)dx + \int_2^0 f(x)dx =$

$\int_1^2 f(x)\,dx - \int_0^2 f(x)\,dx$

$3-5 = -2$

10. $\int_1^2 g(x)\,dx + \int_2^0 g(x)dx =$

$5-4 = 1$

For 11 and 12 Rewrite the following as a single integral

11. $\int_1^3 f(x)dx + \int_3^6 f(x)dx + \int_6^{12} f(x)dx$

$\int_1^{12} f(x)\,dx$

12. $\int_2^{10} f(x)dx - \int_2^7 f(x)dx$

$\int_7^{10} f(x)\,dx$

13. If $\int_2^8 f(x)dx = 1.7$ and $\int_5^8 f(x)dx = 2.5$, find $\int_2^5 f(x)dx$

$\int_2^5 f(x)\,dx + \int_5^8 f(x)\,dx = \int_2^8 f(x)\,dx$

$\int_2^5 f(x)dx + 2.5 = 1.7$

$\boxed{\int_2^5 f(x)\,dx = -0.8}$

14. If $\int_0^1 f(t)dt = 2$, $\int_0^4 f(t)dt = -6$ and $\int_3^4 f(t)dt = 1$ find $\int_1^3 f(t)dt$

$\int_0^1 f(t)\,dt + \int_1^3 f(t)\,dt + \int_3^4 f(t)\,dt = \int_0^4 f(t)\,dt$

$2 + \int_1^3 f(t)\,dt + 1 = -6$

$\boxed{\int_1^3 f(t)\,dt = -9}$

15. If $\int_0^1 x^2 dx = \frac{1}{3}$ use this to determine $\int_0^1 (5-6x^2)dx$

$\int_0^1 5\,dx - \int_0^1 6x^2\,dx$

$\int_0^1 5\,dx - 6\int_0^1 x^2\,dx$

$5x\Big]_0^1 - 6[\tfrac{1}{3}] \Rightarrow (5-0) - 2 = 3$

16. If $\int_1^3 e^x dx = e^3 - e$ use this to determine $\int_1^3 (2e^x - 1)dx$

$2\int_1^3 e^x\,dx - \int_1^3 1\,dx$

$2(e^3 - e) - x\Big]_1^3$

$(-3-1)$

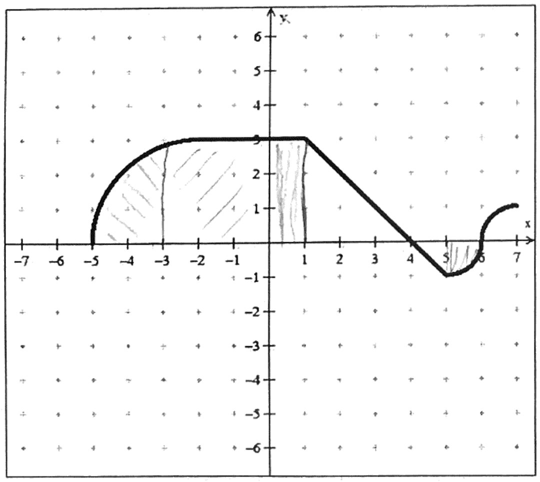
$\boxed{2e^3 - 2e + 4}$

17. If $\int_1^3 e^x dx = e^3 - e$ use this to determine $\int_1^3 e^{x+2} dx$

$\int_1^3 e^x e^2\,dx$

$e^2 \int_1^3 e^x\,dx = e^2(e^3 - e) = e^5 - e^3$

Use the graph below to answer the following questions.



1. $\int_{-5}^{-2} f(x)dx$ $\quad \frac{1}{4}\pi r^2$

$\frac{1}{4}\pi(3)^2 = \frac{9\pi}{4}$

2. $\int_{-5}^{0} f(x)dx$ $\quad \int_{-5}^{-2} f(x)\,dx + \int_{-2}^{0} f(x)\,dx$

$\frac{9\pi}{4} + (3)(3)$

$\frac{9\pi}{4} + 9 = \frac{9\pi + 36}{4}$

3. $\int_0^5 f(x)dx$

$\int_0^1 f(x)\,dx + \int_1^4 f(x)\,dx + \int_4^5 f(x)\,dx$

$1(3) + \frac{1}{2}(3)(3) + \frac{1}{2}(1)(-1) = 3 + \frac{9}{2} - \frac{1}{2} = 7$

4. $\int_5^7 f(x)dx$

$\int_5^6 f(x)\,dx + \int_6^7 f(x)\,dx$

$-\frac{1}{4}\pi(1)^2 + \frac{1}{4}(\pi)(1)^2 = 0$

5. $\int_5^7 |f(x)|dx$

$\left|-\frac{1}{4}\pi(1)^2\right| + \frac{1}{4}\pi(1)^2 = \frac{\pi}{2}$

6. $\int_{-2}^{1} (f(x) + 2)\,dx$

$\int_{-2}^1 f(x)\,dx + \int_{-2}^1 2\,dx$

$3(3) + 2x\Big]_{-2}^1 = 9 + [2 - (-4$

$9 + 6$

15

7. $\int_1^4 5f(x)dx$

$5\int_1^4 f(x)\,dx$

$5(\frac{1}{2}(3)(3)) = \frac{45}{2}$

8. $\int_4^{-5} f(x)dx$

$-\int_{-5}^{4} f(x)\,dx$

$-[\int_{-5}^0 f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^4 f(x$

$-[\frac{9\pi}{4} + 9 + (1)(3) + \frac{1}{2}(3)(3)]$

$-[\frac{9\pi}{4} + \ldots = \frac{-9\pi - 66}{\ldots}$