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The Influence of Statistical Normalization Techniques on Performance Ranking Results:

The Application of MCDM Method Proposed by Biswas and Saha

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ABSTRACT

In this study, the most suitable normalization techniques for the multi-criteria decision-making (MCDM) method proposed by Biswas and Saha were compared and a real situation was analyzed. In the study, the financial performance of the top 10 companies on the Fortune 500 list for 2019 was evaluated using seven financial ratios and five well-known normalization techniques. The results have shown that the max normalization procedure generated the most consistent results for Biswas and Saha's MCDM method. The study is the first to test the suitability of different normalization techniques for the MCDM method proposed by Biswas and Saha. Also, this paper provides decision support that can be used for the selection of the best normalization techniques for other MCDM methods.

KEYWORDS

Consistency, MCDM, MCDM Method Proposed by Biswas and Saha, Normalization, Performance Evaluation

INTRODUCTION

In most cases, choosing between multiple alternatives and criteria is a difficult task for the decision-makers. In such cases, MCDM techniques provide a convenient way for decision-makers to reach a solution. In MCDM models, each alternative has a performance rating for each attribute, and performance ratings for different attributes are usually measured by different units. Therefore, normalization procedures are used to transform different units of measure into comparable units in MCDM models (Celen, 2014, p. 186).

The first step in most MCDM methods is the normalization procedure. Different normalization techniques can have different effects on the ranking results. This causes deviation from the optimal ranking. Therefore, the selection of suitable normalization techniques plays an significant role on the final results of the decision problems (Vafaei et al., 2020, p. 43).

The effects of different normalization techniques on MCDM methods have been investigated in various studies in the literature. Gardziejczyk & Zabicki (2017) dealt with the selection of road alignment variant problem and compared the results obtained using eleven different normalization techniques. They concluded that Van Delft & Nijkamp, linear and pattern normalization techniques generated the most consistent results. Migilinskas & Ustinovichius (2007) examined the effects of eight popular normalization procedures in their studies. It was concluded that proximity to ideal point

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and sum normalization techniques gave healthy results in the presence of five or fewer alternatives. Chakraborty & Yeh (2009) tested the effect of normalization techniques for the Simple Additive Weighting (SAW) method. It was shown that vector normalization was more appropriate than other procedures. Vafaei et al. (2020) tested the suitability of four different normalization techniques for the Analytical Hierarchy Process (AHP) method. It was determined that the best normalization technique for the AHP method is the max-min. Jafaryeganeh et al. (2020) measured the effect of four different normalization techniques for Weighted Sum Method (WSM), Weighted Product Method (WPM), Technique for Order Preferences by Similarity to an Ideal Solution (TOPSIS) and Elimination et Choice Translating Reality (ELECTRE) methods. It was found that normalization techniques that can preserve the dominance order of the alternatives result in a similar final design choice. Aytekin (2021) used fourteen sets representing different decision problem scenarios to compare different normalization techniques. It was concluded that the decision-maker chooses the alternative with the highest value in the criteria or, on the contrary, optimization-based normalization techniques should be preferred. Reference-based normalization techniques are considered suitable for situations where ideal values determined by the decision maker for each criterion. Brauers & Zavadskas (2006) proved the appropriateness of five normalization procedures for the Multi-objective Optimization By Ratio Analysis (MOORA) method. It was determined that the vector normalization technique produced the most consistent results. Milani et al. (2005) used different normalization procedures for the TOPSIS method. The results showed that linear normalization techniques did not significantly affect the variant ranking. Yazdani et al. (2017) measured the effects of different normalization techniques for the COmplex PRoportional ASsessment (COPRAS)-G model. It was determined that the number of criteria and alternatives affected the ranking results. Peldschus (2007) compared different normalization procedures and determined that normalization procedures had an effect on the MCDM results. It was also concluded that a linear normalization cannot be used to solve maximization or minimization problems. Kosareva et al. (2018) used the various normalization techniques to test the SAW method. They found that of all five techniques, none were the best or worst in all cases and the logarithmic normalization technique was the worst in some cases. Vafaei et al. (2016) examined the effects of the most suitable techniques for the Analytical Hierarchy Process (AHP) method. It was revealed that the logarithmic normalization technique could not be used in the AHP method, asit leads to zero or infinite values in normalized data. Celen (2014) tested the various normalization procedures for the FAHP and TOPSIS. It was determined that the most coherent results were obtained by vector normalization. Ersoy (2021) tested the suitability of eight normalization techniques for the ROV method. It was determined that non-linear normalization was the most suitable technique for ROV method.

In this study, real life application was carried out by focusing on the effects of different normalization techniques on the MCDM method results. Accordingly, the MCDM method proposed by Biswas and Saha was used to evaluate the financial performance of the top 10-ranked companies in the FORTUNE 500 list by 2019. Also, the suitability of five different normalization techniques for this method was tested.

The significance of the proposed model are as follows:

- This is the first study to test the effect of different normalization techniques for the MCDM method proposed by Biswas and Saha.
- It is also the first study that is used the MCDM method proposed by Biswas and Saha for measuring the financial performance.
- It is hoped that this study will motivate and guide researchers to try similar applications for different MCDM methods.
- The results obtained in the study with five different normalization techniques are considered essential for being comparable.

The rest of this paper is organized as follows: In Section 2, normalization techniques used in the application are described. In section 3, the MCDM method used in the study is explained. The real case application is given in Section 4. Finally, the concluding remarks and future research directions are given in Section 5.

NORMALIZATION TECHNIQUES

Normalization techniques can be divided into classes in various ways. In this study, Jüttler's (1966), Körth's (1969) and Logarithmic (Zavadskas & Turskis, 2008) normalization techniques were not addressed because they caused negative values in the normalized decision matrix. The characteristics of the normalization techniques used in this study are presented below.

Vector Normalization Technique

The vector normalization technique was introduced by Van Delft & Nijkamp (1977). The formulation of the technique is as follows.

$$w_j = \frac{1}{n} \text{ for benefit criteria} \tag{1}$$

$$r_{ij} = \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^{2}}} \text{ for cost criteria} \tag{2}$$

Max Normalization Technique

Max normalization technique was introduced by Stopp (1975). The formulation of the technique is as follows.

$$r_{ij} = 1 - \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^{2}}} \text{ for benefit criteria} \tag{3}$$

$$r_{ij} = \frac{r_{ij}}{\max\limits_{1 \le i \le m} r_{ij}} \text{ for cost criteria} \tag{4}$$

Sum Normalization Technique

Sum normalization technique was introduced by Wang & Luo (2010). The formulation of the technique is as follows.

$$r_{ij} = \frac{\min\limits_{1 \le i \le m} r_{ij}}{r_{ij}} \text{ for benefit criteria} \tag{5}$$

$$r_{ij} = \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}} \text{ for cost criteria} \tag{6}$$

Minmax Normalization Technique

The Minmax normalization technique was introduced by Weitendorf (1976). The formulation of the technique is as follows.

$$r_{ij} = \frac{1 / r_{ij}}{\sum_{i=1}^{m} 1 / r_{ij}} \text{ for benefit criteria} \tag{7}$$

$$r_{ij} = \frac{r_{ij} - \min\limits_{1 \le i \le m} r_{ij}}{\max\limits_{1 \le i \le m} r_{ij} - \min\limits_{1 \le i \le m} r_{ij}} \text{ for cost criteria} \tag{8}$$

Peldschus Normalization Technique

This normalization technique was introduced by Peldschus (1986). The formulation of the technique is as follows.

$$r_{ij} = \frac{\max\limits_{1 \le i \le m} r_{ij} - r_{ij}}{\max\limits_{1 \le i \le m} r_{ij} - \min\limits_{1 \le i \le m} r_{ij}} \text{ for benefit criteria} \tag{9}$$

$$x_{ij}^{\ *} = (x_{ij} \ / \ x_j^{\ +})^2 \text{ for cost criteria} \tag{10}$$

THE MCDM METHOD PROPOSED BY BISWAS AND SAHA

This method was proposed by Biswas & Saha (2019) to be applied in decision-making processes. It has advantages over other MCDM methods due to the simplicity of its implementation steps and its solid mathematical basis. Its other advantage is that a small number of researchers have handled it. The formulation of the method is as follows (Biswas & Saha, 2019, pp. 139-140).

Step 1: Construction of a decision matrix. A decision matrix is created.

Step 2: Construction of the normalized decision matrix $x_{ij}^{*} = (x_{j}^{-} \ / \ x_{ij})^{3}$

Benefit-oriented and cost-oriented criteria are normalized using equations (11), and (12), respectively.

$$N = \left[r_{ij}\right]_{m*n} \text{ for benefit criteria} \tag{11}$$

$$r_{ij} = \frac{x_{ij} - x_{i}^{-}}{x_{i}^{+} - x_{i}^{-}} \text{ for cost criteria} \tag{12}$$

x_{ij} value of the i-th variant according to j-th criterion
$x_j^+ = \max(x_{ij})$; $x_j^- = \min(x_{ij})$

Step 3: Construction of the scaled normalized decision matrix

if the normalized value is in the range (>0.80, 1.00], the scale factor is taken as g = 5. if the normalized value is in the range (>0.60, 0.80), g =4, if the normalized value is in the range (>0.40, 0.60), the scale factor g=3, (>0.20.40) scale factor 2 and (>0.0.20) g=1.

Step 4: Calculation of the weighted normalized matrix (V)

Using equation (13), normalized matrix elements are multiplied by criterion weights and weighted normalized matrix (V) are calculated.

$$r_{ij} = \frac{x_{ij} - x_{i}^{+}}{x_{i}^{-} - x_{i}^{+}} \tag{13}$$

Step 5: Computing of overall score of the alternatives

The overall score of the alternatives is calculated using equation (14). Alternatives are ranked according to the decreasing value of Si.

$$q_{ij} = w_{i} * v_{ij} \tag{14}$$

A REAL CASE STUDY

In this study, the suitability of five different normalization techniques was tested for the MCDM method proposed by Biswas and Saha. For this purpose, the financial performances of the top 10 companies in the FORTUNE 500 list in 2019 were evaluated using the MCDM method on the basis of seven financial ratios, and a real-life application was presented. Accordingly, companies included in the study are presented in Table 1.

The seven financial ratios (criteria) are given in Table 2.

Table 1. Firms within the scope of the study

Rank	Company's Name
1	"Walmart"
2	"Exxon Mobil"
3	"Apple"
4	"Berkshire Hathaway"
5	"Amazon.com"
6	"UnitedHealth Group"
7	"McKesson"
8	"CVS Health"
9	"AT&T"
10	"AmerisourceBergen"

Table 2. Financial ratios

Rank	Code	"Financial Ratios and Disclosures"	
"Liquidity ratios"			"Opt."
1	CR	"Current ratio= Current Assets / Current Liabilities"	"max"
2	QR	"Quick ratio = (Current Assets - Inventories) / Current Liabilities"	"max"
"Leverage ratios"			
3	FR	"Financing Rate = Total Shareholders' Equity/ Total Liabilities"	"min"
4	LR	"Leverage Ratio = Total Liabilities /Total assets"	"min"
"Profitability ratios"			
5	NPM	"Net Profit Margin Ratio= Net Income / Net sales"	*"max"*
6	ROA	"Return on Assets = Net Income (annual)/ Total assets"	*"max"*
" Efficiency Ratios"			
7	ATR	"Asset Turnover Rate = Net Sales/Total Assets"	"max"

WEIGHTING OF CRITERIA

With the help of the formula (15) (Jahan et al., 2012, p. 413), an equal weight was assigned to each criterion during the application (15)

n indicates the number of criteria and the sum of weights should be equal to 1.

The criteria weights are presented in Table 3.

Table 3. Criteria weights

CR	QR	FR	LR	NPM	ROA	ATR
0.142857	0.142857	0.142857	0.142857	0.142857	0.142857	0.142857

APPLICATION OF THE MCDM METHOD PROPOSED BY BISWAS AND SAHA FOR FINANCIAL PERFORMANCE EVALUATION OF FIRMS

The steps followed in evaluating the performance of the companies using the MCDM method proposed by Biswas and Saha are given below.

Step 1: As a first step, the decision matrix is created and presented in Table 4.

Table 4. Decision matrix

“Alternatives	Criteria						
	CR	QR	FR	LR	NPM	ROA	ATR
“Walmart”	0.80	0.23	0.57	0.64	0.01	0.03	2.33
“Exxon Mobil”	0.78	0.56	1.22	0.45	0.06	0.04	0.70
Apple	1.54	1.50	0.36	0.73	0.21	0.16	0.77
Berkshire Hathaway	1.76	1.46	1.09	0.48	0.32	0.10	0.31
Amazon.com	0.41	0.18	0.38	0.72	0.04	0.05	1.25
“UnitedHealth Group”	0.69	0.58	0.54	0.64	0.06	0.08	1.39
“McKesson”	1.02	0.58	0.16	0.86	0.001	0.004	3.59
CVS Health	0.94	0.62	0.41	0.71	0.03	0.03	1.15
AT&T	0.79	0.50	0.58	0.63	0.08	0.03	0.33
AmerisourceBergen	0.95	0.58	0.08	0.92	0.01	0.02	4.58

Step 2: In the normalization phase, five different techniques are used and the process steps are detailed below.

In order to normalize the decision matrix with the Vector normalization technique;
The normalization calculation of Walmart's CR criterion is as follows.
For benefit criteria;

$$S_i = \sum_{j=1}^{n} q_{ij}$$

The normalization calculation of Walmart's LR criterion is as follows.
For cost criteria;

$$\frac{0.8}{\sqrt{(0.8)^2+(0.78)^2+(1.54)^2+(1.76)^2+(0.41)^2+(0.69)^2+(1.02)^2+(0.94)^2+(0.79)^2+(0.95)^2}} = 0.243$$

Table 5. Normalization matrix (Vector normalization)

	CR	QR	FR	LR	NPM	ROA	ATR
Walmart	0.243	0.091	0.719	0.708	0.025	0.135	0.346
“Exxon Mobil”	0.237	0.221	0.398	0.794	0.149	0.180	0.104
“Apple”	0.468	0.592	0.822	0.666	0.520	0.721	0.114
Berkshire Hathaway	0.535	0.576	0.462	0.781	0.793	0.451	0.046
Amazon.com	0.125	0.071	0.812	0.671	0.099	0.225	0.186
“UnitedHealth Group”	0.210	0.229	0.733	0.708	0.149	0.361	0.206
“McKesson”	0.310	0.229	0.921	0.607	0.002	0.018	0.533
CVS Health	0.286	0.245	0.798	0.676	0.074	0.135	0.171
AT&T	0.240	0.197	0.714	0.712	0.198	0.135	0.049
AmerisourceBergen	0.289	0.229	0.961	0.580	0.025	0.090	0.680

The results obtained are given in Table 5.
In order to normalize the decision matrix with the Max normalization technique;
The normalization calculation of Walmart's CR criterion is as follows.

For benefit criteria;

$$1-\frac{0.64}{\sqrt{(0.64)^2+(0.45)^2+(0.73)^2+(0.48)^2+(0.72)^2+(0.64)^2+(0.86)^2+(0.71)^2+(0.63)^2+(0.92)^2}} = 0.708$$

The normalization calculation of Walmart's LR criterion is as follows.

For cost criteria; $\frac{0.8}{1.76} = 0.455$

Table 6. Normalization matrix (Max normalization)

	CR	QR	FR	LR	NPM	ROA	ATR
Walmart	0.455	0.153	0.140	0.703	0.031	0.188	0.509
Exxon Mobil	0.443	0.373	0.066	1	0.188	0.250	0.153
Apple	0.875	1	0.222	0.616	0.656	1	0.168
Berkshire Hathaway	1	0.973	0.073	0.938	1	0.625	0.068
Amazon.com	0.233	0.12	0.211	0.625	0.125	0.313	0.273
"UnitedHealth Group"	0.392	0.387	0.148	0.703	0.188	0.5	0.303
"McKesson"	0.580	0.387	0.5	0.523	0.003	0.025	0.784
CVS Health	0.534	0.413	0.195	0.634	0.094	0.188	0.251
AT&T	0.449	0.333	0.138	0.714	0.250	0.188	0.072
AmerisourceBergen	0.540	0.387	1	0.489	0.031	0.125	1

The results obtained are given in Table 6.
In order to normalize the decision matrix with the Sum normalization technique;
The normalization calculation of Walmart's CR criterion is as follows.

For benefit criteria; $\frac{0.45}{0.64} = 0.703$

Table 7. Normalization matrix (Sum normalization)

	CR	QR	FR	LR	NPM	ROA	ATR
Walmart	0.083	0.034	0.052	0.101	0.012	0.055	0.142
"Exxon Mobil"	0.081	0.082	0.024	0.144	0.073	0.074	0.043
"Apple"	0.159	0.221	0.083	0.089	0.256	0.294	0.047
Berkshire Hathaway	0.182	0.215	0.027	0.135	0.390	0.184	0.019
"Amazon.com"	0.042	0.027	0.078	0.090	0.049	0.092	0.076
UnitedHealth Group	0.071	0.085	0.055	0.101	0.073	0.147	0.085
McKesson	0.105	0.085	0.186	0.075	0.001	0.007	0.219
"CVS Health"	0.097	0.091	0.072	0.091	0.037	0.055	0.070
AT&T	0.082	0.074	0.051	0.103	0.097	0.055	0.020
AmerisourceBergen	0.098	0.085	0.371	0.070	0.012	0.037	0.279

The normalization calculation of Walmart's LR criterion is as follows. The results obtained are given in Table 7.

For cost criteria; $\dfrac{0.8}{9.68} = 0.083$

In order to normalize the decision matrix with the Minmax normalization technique;
The normalization calculation of Walmart's CR criterion is as follows.

For benefit criteria; $\dfrac{1\,/\,0.64}{15.43} = 0.101$

The normalization calculation of Walmart's LR criterion is as follows. The results obtained are given in Table 8.

For cost criteria; $\dfrac{0.8 - 0.41}{1.76 - 0.41} = 0.289$

Table 8. Normalization matrix (Minmax normalization)

	CR	QR	FR	LR	NPM	ROA	ATR
Walmart	0.289	0.038	0.570	0.596	0.028	0.167	0.473
Exxon Mobil	0.274	0.288	0	1	0.185	0.231	0.091
Apple	0.837	1	0.754	0.404	0.655	1	0.108
Berkshire Hathaway	1	0.970	0.114	0.936	1	0.615	0
Amazon.com	0	0	0.737	0.426	0.122	0.295	0.220
"UnitedHealth Group"	0.207	0.303	0.596	0.596	0.185	0.487	0.253
"McKesson"	0.452	0.303	0.930	0.128	0	0	0.768
CVS Health	0.393	0.333	0.711	0.447	0.091	0.167	0.197
AT&T	0.281	0.242	0.561	0.617	0.248	0.167	0.005
AmerisourceBergen	0.4	0.303	1	0	0.028	0.103	1

In order to normalize the decision matrix with the Peldschus normalization technique;
The normalization calculation of Walmart's CR criterion is as follows.

For benefit criteria; $\dfrac{0.64 - 0.92}{0.45 - 0.92} = 0.596$

The normalization calculation of Walmart's LR criterion is as follows. The results obtained are given in Table 9.

For cost criteria; $(\frac{0.8}{1.76})^2 = 0.207$

Step 3: In this step, Scaled Normalized decision matrix is created. All the results obtained are given below.

Table 9. Normalization matrix (Peldschus normalization)

	CR	QR	FR	LR	NPM	ROA	ATR
Walmart	0.207	0.024	0.003	0.348	0.001	0.035	0.259
Exxon Mobil	0.196	0.139	0.0003	1	0.035	0.063	0.023
Apple	0.766	1	0.011	0.234	0.431	1	0.028
Berkshire Hathaway	1	0.947	0.0004	0.824	1	0.391	0.005
Amazon.com	0.054	0.014	0.009	0.244	0.016	0.098	0.074
"UnitedHealth Group"	0.154	0.150	0.003	0.348	0.035	0.25	0.092
"McKesson"	0.336	0.150	0.125	0.143	0.00001	0.0006	0.614
CVS Health	0.285	0.171	0.007	0.255	0.009	0.035	0.063
AT&T	0.201	0.111	0.003	0.364	0.063	0.035	0.005
AmerisourceBergen	0.291	0.150	1	0.117	0.001	0.016	1

Step 4: In this step, weighted normalized matrix (V) is calculated using the weights in Table 3 and all the results obtained are given below.

Table 10. Scaled normalized decision matrix (Vector normalization)

	CR	QR	FR	LR	NPM	ROA	ATR
Walmart	2	1	4	4	1	1	2
Exxon Mobil	2	2	2	4	1	1	1
Apple	3	3	5	4	3	4	1
Berkshire Hathaway	3	3	3	4	4	3	1
Amazon.com	1	1	5	4	1	2	1
UnitedHealth Group	2	2	4	4	1	2	2
McKesson	2	2	5	4	1	1	3
CVS Health	2	2	4	4	1	1	1
AT&T	2	1	4	4	1	1	1
AmerisourceBergen	2	2	5	3	1	1	4

Table 11. Scaled normalized decision matrix (Max normalization)

	CR	QR	FR	LR	NPM	ROA	ATR
Walmart	3	1	1	4	1	1	3
Exxon Mobil	3	2	1	5	1	2	1
Apple	5	5	2	4	4	5	1
Berkshire Hathaway	5	5	1	5	5	4	1
Amazon.com	2	1	2	4	1	2	2
UnitedHealth Group	2	2	1	4	1	3	2
McKesson	3	2	3	3	1	1	4
CVS Health	3	3	1	4	1	1	2
AT&T	3	2	1	4	2	1	1
AmerisourceBergen	3	2	5	3	1	1	5

Table 12. Scaled normalized decision matrix (Sum normalization)

	CR	QR	FR	LR	NPM	ROA	ATR
Walmart	1	1	1	1	1	1	1
Exxon Mobil	1	1	1	1	1	1	1
Apple	1	2	1	1	2	2	1
Berkshire Hathaway	1	2	1	1	2	1	1
Amazon.com	1	1	1	1	1	1	1
UnitedHealth Group	1	1	1	1	1	1	1
McKesson	1	1	1	1	1	1	2
CVS Health	1	1	1	1	1	1	1
AT&T	1	1	1	1	1	1	1
AmerisourceBergen	1	1	2	1	1	1	2

All results obtained by five different normalization techniques are given in Table 20.

In Table 20, the ranking results obtained with the five normalization techniques were different from each other. While Apple company ranked first according to the results obtained with four normalization techniques (Vector, Max, Sum, Minmax), it took second according to the Peldschus normalization technique. On the other hand, Walmart, Exxon Mobil, Amazon.com, United Health Group, CVS Health and AT&T companies ranked last. Another result that attracts attention is that some companies share the same order.

According to figure 1, the direction of change of the rankings obtained with five different normalization techniques are quite similar, but the rankings of the companies differed from each other with small deviations. At this stage, it is difficult to decide which normalization technique is best and the worst for the MCDM method proposed by Biswas and Saha.

Table 13. Scaled normalized decision matrix (Minmax normalization)

	CR	QR	FR	LR	NPM	ROA	ATR
Walmart	2	1	3	3	1	1	3
Exxon Mobil	2	2	1	5	1	2	1
Apple	5	5	4	2	4	5	1
Berkshire Hathaway	5	5	1	5	5	4	1
Amazon.com	1	1	4	3	1	2	2
UnitedHealth Group	2	2	3	3	1	3	2
McKesson	3	2	5	1	1	1	4
CVS Health	2	2	4	3	1	1	1
AT&T	2	2	3	4	2	1	1
AmerisourceBergen	2	2	5	1	1	1	5

Table 14. Scaled normalized decision matrix (Peldschus normalization)

	CR	QR	FR	LR	NPM	ROA	ATR
Walmart	2	1	1	2	1	1	2
Exxon Mobil	1	1	1	5	1	1	1
Apple	4	5	1	2	3	5	1
Berkshire Hathaway	5	5	1	5	5	2	1
Amazon.com	1	1	1	2	1	1	1
UnitedHealth Group	1	1	1	2	1	2	1
McKesson	2	1	1	1	1	1	4
CVS Health	2	1	1	2	1	1	1
AT&T	1	1	1	2	1	1	1
AmerisourceBergen	2	1	5	1	1	1	5

Table 15. Weighted normalized matrix (Vector normalization)

	CR	QR	FR	LR	NPM	ROA	ATR	å	Rank
Walmart	0.286	0.143	0.571	0.571	0.143	0.143	0.286	2.143	5
Exxon Mobil	0.286	0.286	0.286	0.571	0.143	0.143	0.143	1.857	7
Apple	0.429	0.429	0.714	0.571	0.429	0.571	0.143	3.286	1
Berkshire Hathaway	0.429	0.429	0.429	0.571	0.571	0.429	0.143	3.000	2
Amazon.com	0.143	0.143	0.714	0.571	0.143	0.286	0.143	2.143	5
UnitedHealth Group	0.286	0.286	0.571	0.571	0.143	0.286	0.286	2.429	4
McKesson	0.286	0.286	0.714	0.571	0.143	0.143	0.429	2.571	3
CVS Health	0.286	0.286	0.571	0.571	0.143	0.143	0.143	2.143	5
AT&T	0.286	0.143	0.571	0.571	0.143	0.143	0.143	2.000	6
AmerisourceBergen	0.286	0.286	0.714	0.429	0.143	0.143	0.571	2.571	3

Table 16. Weighted normalized matrix (Max normalization)

	CR	QR	FR	LR	NPM	ROA	ATR	å	Rank
Walmart	0.429	0.143	0.143	0.571	0.143	0.143	0.429	2.000	5
Exxon Mobil	0.429	0.286	0.143	0.714	0.143	0.286	0.143	2.143	4
Apple	0.714	0.714	0.286	0.571	0.571	0.714	0.143	3.714	1
Berkshire Hathaway	0.714	0.714	0.143	0.714	0.714	0.571	0.143	3.714	1
Amazon.com	0.286	0.143	0.286	0.571	0.143	0.286	0.286	2.000	5
UnitedHealth Group	0.286	0.286	0.143	0.571	0.143	0.429	0.286	2.143	4
McKesson	0.429	0.286	0.429	0.429	0.143	0.143	0.571	2.429	3
CVS Health	0.429	0.429	0.143	0.571	0.143	0.143	0.286	2.143	4
AT&T	0.429	0.286	0.143	0.571	0.286	0.143	0.143	2.000	5
AmerisourceBergen	0.429	0.286	0.714	0.429	0.0,143	0.143	0.714	2.857	2

Table 17. Weighted normalized matrix (Sum normalization)

	CR	QR	FR	LR	NPM	ROA	ATR	å	Rank
Walmart	0.143	0.143	0.143	0.143	0.143	0.143	0.143	1.000	4
Exxon Mobil	0.143	0.143	0.143	0.143	0.143	0.143	0.143	1.000	4
Apple	0.143	0.286	0.143	0.143	0.286	0.286	0.143	1.429	1
Berkshire Hathaway	0.143	0.286	0.143	0.143	0.286	0.143	0.143	1.286	2
Amazon.com	0.143	0.143	0.143	0.143	0.143	0.143	0.143	1.000	4
UnitedHealth Group	0.143	0.143	0.143	0.143	0.143	0.143	0.143	1.000	4
McKesson	0.143	0.143	0.143	0.143	0.143	0.143	0.286	1.143	3
CVS Health	0.143	0.143	0.143	0.143	0.143	0.143	0.143	1.000	4
AT&T	0.143	0.143	0.143	0.143	0.143	0.143	0.143	1.000	4
AmerisourceBergen	0.143	0.143	0.286	0.143	0.143	0.143	0.286	1.286	2

Table 18. Weighted normalized matrix (Minmax normalization)

	CR	QR	FR	LR	NPM	ROA	ATR	å	Rank
Walmart	0.286	0.143	0.429	0.429	0.143	0.143	0.429	2.000	5
Exxon Mobil	0.286	0.286	0.143	0.714	0.143	0.286	0.143	2.000	5
Apple	0.714	0.714	0.571	0.286	0.571	0.714	0.143	3.714	1
Berkshire Hathaway	0.714	0.714	0.143	0.714	0.714	0.571	0.143	3.714	1
Amazon.com	0.143	0.143	0.571	0.429	0.143	0.286	0.286	2.000	5
UnitedHealth Group	0.286	0.286	0.429	0.429	0.143	0.429	0.286	2.286	3
McKesson	0.429	0.286	0.714	0.143	0.143	0.143	0.571	2.429	2
CVS Health	0.286	0.286	0.571	0.429	0.143	0.143	0.143	2.000	5
AT&T	0.286	0.286	0.429	0.571	0.286	0.143	0.143	2.143	4
AmerisourceBergen	0.286	0.286	0.714	0.143	0.143	0.143	0.714	2.429	2

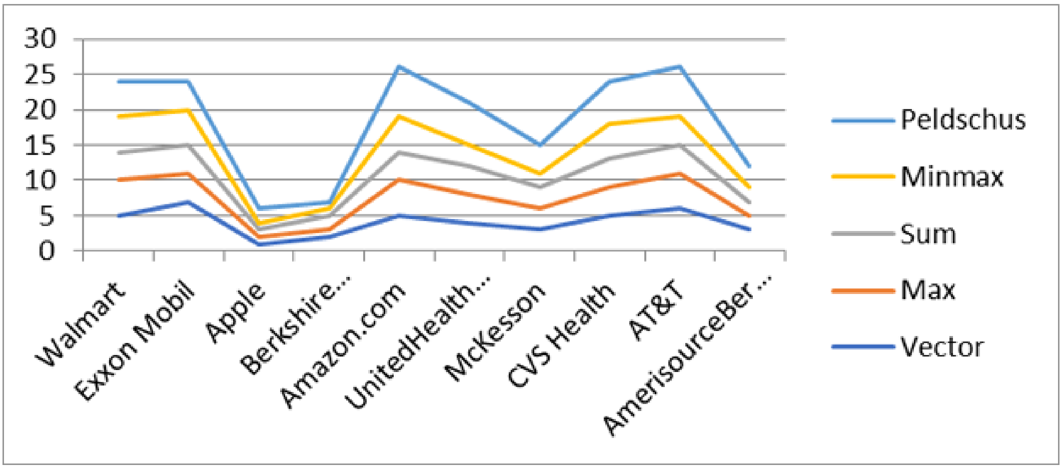
Table 19. Weighted normalized matrix (Peldschus normalization)

	CR	QR	FR	LR	NPM	ROA	ATR	å	Rank
Walmart	0.286	0.143	0.143	0.286	0.143	0.143	0.286	1.429	5
Exxon Mobil	0.143	0.143	0.143	0.714	0.143	0.143	0.143	1.571	4
Apple	0.571	0.714	0.143	0.286	0.429	0.714	0.143	3	2
Berkshire Hathaway	0.714	0.714	0.143	0.714	0.714	0.286	0.143	3.429	1
Amazon.com	0.143	0.143	0.143	0.286	0.143	0.143	0.143	1.143	7
UnitedHealth Group	0.143	0.143	0.143	0.286	0.143	0.286	0.143	1.286	6
McKesson	0.286	0.143	0.143	0.143	0.143	0.143	0.571	1.571	4
CVS Health	0.286	0.143	0.143	0.286	0.143	0.143	0.143	1.286	6
AT&T	0.143	0.143	0.143	0.286	0.143	0.143	0.143	1.143	7
AmerisourceBergen	0.286	0.143	0.714	0.143	0.143	0.143	0.714	2.286	3

Table 20. Ranking results

	Normalization Techniques				
	Vector	Max	Sum	Minmax	Peldschus
Walmart	5	5	4	5	5
Exxon Mobil	7	4	4	5	4
Apple	1	1	1	1	2
Berkshire Hathaway	2	1	2	1	1
Amazon.com	5	5	4	5	7
UnitedHealth Group	4	4	4	3	6
McKesson	3	3	3	2	4
CVS Health	5	4	4	5	6
AT&T	6	5	4	4	7
AmerisourceBergen	3	2	2	2	3

Figure 1. Ranking results



ASSESSMENT APPROACH OF NORMALIZATION TECHNIQUES FOR THE MCDM METHOD PROPOSED BY BISWAS AND SAHA

In the literature, there are several approaches used to test the suitability of different normalization techniques for MCDM techniques. For example, Chakraborty & Yeh (2009) developed a ranking consistency index (RCI) to determine how a particular normalization procedure produces similar sequences with other procedures. Celen (2014) used a measure of consistency containing the Pearson correlations to assess the normalization techniques for assessing the performance of the banks in Turkey.

In this study, the approaches proposed by Chakraborty & Yeh (2009) and Celen (2014) were used to test the effectiveness of normalization techniques. In addition to these two approaches, Spearman's approach (Wang & Luo, 2010) was used. The 2-step process that includes all three approaches is given below.

Step A: In this step, RCI application is included. Since we have 5 normalization techniques, the consistency weight (CW) is used as follows:

1) "If a technique is consistent with all other four techniques", then CW = 4/4 = 1
2) "If a technique is consistent with three of the four techniques", then CW = 3/4
3) "If a technique is consistent with two of the four techniques", then CW = 2/4
4) "If a technique is consistent with one of the four techniques", then CW = 1/4
5) "If a technique is not consistent with any other five techniques", then CW = 0/4 = 0.

The RCI of **Vector** normalization techniques is calculated as;

$$\text{RCI (Vector)} = [(T_{12345}*(CW=1))+(T_{1234}*(CW=3/4))+(T_{1245}*(CW=3/4))+(T_{1235}*(CW=3/4))+(T_{1345}*(CW=3/4))+(T_{123}*(CW=2/4))+(T_{124}*(CW=2/4))+T_{125}*(CW=2/4))+(T_{134}*(CW=2/4))+(T_{135}*(CW=2/4))+(T_{145}*(CW=2/4))+(T_{12}*(CW=1/4))+(T_{13}*(CW=1/4))+(T_{124}*(CW=1/4))+(T_{15}*(CW=1/4))+(TD_{12345}*(CW=0))/TS]$$

The RCI of the **Max** normalization techniques is calculated as;

RCI (Max) = [(T_{12345}*(CW=1))+(T_{2345}*(CW=3/4))+(T_{2134}*(CW=3/4))+(T_{2145}*(CW=3/4))+(T_{2135}*(CW=3/4))+(T_{213}*(CW=2/4))+(T_{214}*(CW=2/4))+T_{215}*(CW=2/4))+(T_{235}*(CW=2/4))+(T_{234}*(CW=2/4))+(T_{245}*(CW=2/4))+(T_{21}*(CW=1/4))+(T_{23}*(CW=1/4))+(T_{24}*(CW=1/4))+(T_{25}*(CW=1/4))+(TD_{12345}*(CW=0))/TS]

The RCI of the **Sum** normalization techniques is calculated as;

RCI (Sum) = [(T_{12345}*(CW=1))+(T_{3451}*(CW=3/4))+(T_{3452}*(CW=3/4))+(T_{3124}*(CW=3/4))+(T_{3125}*(CW=3/4))+(T_{321}*(CW=2/4))+(T_{324}*(CW=2/4))+T_{325}*(CW=2/4))+(T_{314}*(CW=2/4))+(T_{315}*(CW=2/4))+(T_{345}*(CW=2/4))+(T_{31}*(CW=1/4))+(T_{32}*(CW=1/4))+(T_{34}*(CW=1/4))+(T_{35}*(CW=1/4))+(TD_{12345}*(CW=0))/TS]

The RCI of the **Minmax** normalization techniques is calculated as;
RCI (Minmax) = [(T_{12345}*(CW=1))+(T_{4512}*(CW=3/4))+(T_{4513}*(CW=3/4))+(T_{4523}*(CW=3/4))+(T_{4123}*(CW=3/4))+(T_{451}*(CW=2/4))+(T_{452}*(CW=2/4))+T_{453}*(CW=2/4))+(T_{412}*(CW=2/4))+(T_{413}*(CW=2/4))+(T_{423}*(CW=2/4))+(T_{41}*(CW=1/4))+(T_{42}*(CW=1/4))+(T_{43}*(CW=1/4))+(T_{45}*(CW=1/4))+(TD_{12345}*(CW=0))/TS]
The RCI of the **Peldschus** normalization techniques is calculated as;

RCI (Peldschus) = [(T_{12345}*(CW=1))+(T_{5123}*(CW=3/4))+(T_{5124}*(CW=3/4))+(T_{5134}*(CW=3/4))+(T_{5234}*(CW=3/4))+(T_{534}*(CW=2/4))+(T_{532}*(CW=2/4))+T_{531}*(CW=2/4))+(T_{512}*(CW=2/4))+(T_{514}*(CW=2/4))+(T_{524}*(CW=2/4))+(T_{51}*(CW=1/4))+(T_{52}*(CW=1/4))+(T_{53}*(CW=1/4))+(T_{54}*(CW=1/4))+(TD_{12345}*(CW=0))/TS]

where

RCI(X) "RCI for normalization procedure" (X = N_1, N_2, …, N_5)
TS "Total number of times the simulation was run" (in this study TS = 1)
TD_{12345} "Total number of times N_1, N_2, N_3, N_4, N_5 produced different rankings"
T_{12345} "Total number of times N_1, N_2, N_3, N_4, N_5 produced the same ranking"
T_{1234} "Total number of times N_1, N_2, N_3, N_4 produced the same ranking"
T_{123} "Total number of times N_1, N_2, N_3 produced the same ranking"
T_{12} "Total number of times N_1, N_2 produced the same ranking"

RCI values were calculated for each normalization technique used in the study and the results are presented in Table 21.

Table 21. RCI values and ranking

	RCI	Rank
Vector	9.75	2
Max	10.5	1
Sum	8.5	4
Minmax	9.5	3
Peldschus	4	5

As can be seen from Table 21, the Max normalization technique is in the first place because it has the highest RCI value. In contrast, Peldschus normalization technique is ranked last. Vector normalization technique is in the 2nd place. This technique was followed by the Minmax and Sum normalization techniques, respectively.

Step B: In this step, Pearson correlation (Celen, 2014) and Spearman correlation (Wang & Luo, 2010) were calculated by using ranking results in Table 20. The following formula was used to calculate the Spearman correlation.

$$qs = \left(\frac{0.45}{0.64}\right)^3 = 0.348$$

D_i shows the difference between ranks ri and ri ¢
m represents the number of alternatives
qs value lies between –1 and +1.
All the results obtained are given in Table 22.

Table 22. Correlation values between results

	Vector		Max		Sum		Minmax	
	P*	S**	P*	S**	P*	S**	P*	S**
Vector			.859	.921	.887	.897	.905	.927
Max	.859	.921			.943	.976	.902	.970
Sum	.887	.897	.943	.976			.885	.958
Minmax	.905	.927	.902	.970	.885	.958		
Peldschus	.710	.867	.919	.885	.852	.812	.773	.818
	Peldschus		Mean ks value		Rank			
	P*	S**	P*	S**	P*	S**		
Vector	.710	.867	.840	.903	4	4		
Max	.919	.885	.906	.938	1	1		
Sum	.852	.812	.892	.911	2	3		
Minmax	.773	.818	.866	.918	3	2		
Peldschus			.814	.846	5	5		

*Pearson correlation coefficient **Spearman correlation coefficient

According to Spearman and Pearson correlation results, Max normalization technique was ranked first while Peldschus normalization technique was ranked last.

CONCLUSION

In this study, a real-life application was presented in which the effects of different normalization techniques on the results of MCDM method are discussed. Accordingly, the MCDM method proposed by Biswas and Saha was used to evaluate the financial performance of the top 10 companies in the FORTUNE 500 list by 2019. Also, the suitability of five various normalization techniques (Vector, Max, Sum, Minmax and Peldschus) for this methodwas tested.

A two-step process was followed to evaluate the results obtained with different normalization techniques. First, the RCI method developed by Chakraborty & Yeh (2009) was used to measure the similarity of the rankings obtained with different normalization procedures. In the second step, a consistency measure including Pearson correlation Celen (2014) and Spearman's rank correlation (Wang & Luo, 2010) were used to evaluate normalization techniques.

According to the results obtained with the RCI approach, the Max normalization is the most consistent while Peldschus is the least consistent procedure for the MCDM method proposed by Biswas and Saha. It was determined that the results obtained by Pearson and Spearman rank correlation analysis were similar to RCI results. Overall, the most suitable technique for the MCDM method proposed by Biswas & Saha is Max normalization technique and it is not recommended to use the Peldschus normalization technique. Another remarkable result is that the Max normalization technique is more suitable than the minmax normalization technique in the MCDM method proposed by Biswas and Saha's own algorithm.

Future studies could test the suitability of various normalization techniques for other MCDM methods such as Reference Ideal Method (RIM), Proximity Indexed Value (PIV) method, Range of Value (ROV) Method or similar problem can be handled for different case studies with the results compared. In addition, the criteria weights can be determined by subjective methods such as AHP and Delphi and the results obtained can be compared.

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